Space-time extensions from space-time densities and Bose-Einstein correlations

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Abstract

Using a (3+1)-dimensional solution of the relativistic Euler-equations for $Pb + Pb$ at 160 $AGeV$, space-time extensions of kaon emission zones are calculated from space-time densities and compared to the inverse widths of two-kaon Bose-Einstein correlation functions. The comparison shows a satisfactory agreement and it is concluded that because of the Gaussian shape of the kaon correlation functions, the space-time parameters of the kaon source can be calculated directly from space-time densities. In the case of intensity interferometry of identical pions this simplification is not recommended when applying Gaussian fits because of the present strong effects of resonance decays. The whole discussion is based on the assumption that hadron emission in ultra-relativistic heavy-ion collisions is purely chaotic or that coherence is at least negligible.

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The knowledge about the lifetime and the spatial extensions of hadronic sources in ultrarelativistic nucleus-nucleus collisions constitutes further information in the search for a new state of nuclear matter, the quark-gluon plasma (QGP). In this effort an experimental technique has been developed which is dedicated to the measurement of Bose-Einstein correlations (BEC) [1]. BEC functions of identical bosons are quantum-statistical observables, which in general contain information about ten quantities [2] which characterize a hadron source: lifetimes, longitudinal and transverse extensions of the chaotic and the coherent source, temporal and spatial (longitudinal and transverse) coherence lengths and the chaoticty. In the case of pion interferometry it has been argued that a possible coherent subcomponent of the hadron source is almost inobservable [3] due to the apparently large contributions from resonance decays to BEC. Many parametrizations of experimental BEC functions are based on a neglect of a coherent component, resulting in the description of the hadron emitting source with only three quantities (*e.g.*, lifetime, longitudinal and transverse radii). In ref. [4] it was shown, that in fact even in the case of a purely chaotic hadron source three quantities are not adequate to fully parametrize the system under consideration. *E.g.*, a cylindrically symmetric expanding fireball has in general to be described with an additional "cross-term" radius. Depending on the concepts of introducing a radius or a lifetime into a model of relativistically expanding hadron sources, it appears to be very likely that a direct relation between interferometry radii (better: inverse widths of BEC functions) and the true space-time characteristics of the source can be established.

It is the purpose of this paper to study whether and how the true space-time features of hadron emitting sources are reflected in two-particle Bose-Einstein correlation functions of purely chaotic sources by applying a relativistic hydrodynamical description. Many hydrodynamical models (*cf.* [5] and refs. therein) are available which describe the dynamics of relativistic heavy-ion collisions. HYLANDER [6] belongs to the class of models which apply (3+1)-dimensional relativistic one-fluid-dynamics. It provides fully three-dimensional solutions of the hydrodynamical relativistic Euler-equations [7]. The model has been successfully applied to several different heavy ion collisions at SPS energies [8]. Here some further results of the HYLANDER analysis [5, 9] of $Pb + Pb$ at 160 $AGeV$ (central collisions) will be presented.

From the hydrodynamical analysis [5, 9] of $Pb + Pb$ at 160 $AGeV$, it has been found that the K^- single inclusive spectra consist mainly of directly emitted kaons; less than 10% come from the $K^\star$ resonance. Because the $K^\star$ resonance contributions do not exhibit any significant effect in the inverse widths and the functional shapes of calculated $K^- K^-$ BEC functions, these resonance contributions have been neglected. Introducing the average and the relative four-momentum, $K^\nu \equiv \frac{1}{2}(k_1^\nu + k_2^\nu)$ and $q^\nu \equiv k_1^\nu - k_2^\nu$, of two identical kaons emitted with their individual four-momenta, k_1^ν and k_2^ν, respectively, the Bose-Einstein correlation function of the kaon-pair is in terms of relativistic hydrodynamics defined as (*cf.* refs. [3, 10, 11])

$$C_2(\vec{k}_1, \vec{k}_2) \;=\; 1 \;+\; \frac{|\int d^4x\; g_K^{dir}(x_\nu, K^\nu)\, e^{iq^\nu x_\nu}|^2}{\int d^4x\; g_K^{dir}(x_\nu, k_1^\nu) \cdot \int d^4x\; g_K^{dir}(x_\nu, k_2^\nu)} \;, \tag{1}$$

where $q^0 = E_1 - E_2$, $K^0 = (E_1 + E_2)/2$, and

$$g_K^{dir}(x_\nu, k^\nu) \;=\; \frac{1}{(2\pi)^3} \int_\Sigma \frac{k^\nu d\sigma_\nu(x_\nu')\, \delta^4(x_\nu - x_\nu')}{\exp\left[\dfrac{k^\nu u_\nu(x_\nu') - S\mu_S(x_\nu')}{T_f(x_\nu')}\right] - 1} \;, \qquad x_\nu' \in \Sigma \,. \tag{2}$$

In eq. (2) $d\sigma_\nu$, u_ν and T_f are the differential volume element of the freeze-out hypersurface Σ, the 4-velocity of the fluid and the freeze-out temperature at space-time point x_ν, respectively. The quantity μ_S is the strangeness chemical potential at space-time point x_ν. (For a calculation of BEC of identical pions including the decay of resonances, *cf.* refs. [3, 11].) The function g_K^{dir} can be interpreted as the quantum analogue of the mean number of kaons of four-momentum k^ν at the space-time point x_ν, *i.e.*, one can consider g_K^{dir} as the quantum analogue of the space-time density distribution of the source emitting kaons of fixed four-momenta k^ν. For the following, let us keep the latter interpretation in mind.

Fig. 1 shows a comparison of BEC functions of π^- (solid lines) and of K^- (dashed lines) at $K_\perp = 0$ and $K_\perp = 1$ GeV/c , respectively. The dotted lines correspond to the BEC functions of directly (thermally) produced π^-. It can be seen that BEC functions of π^- are strongly distorted by the

decay of resonances. In particular, the functional shape of the BEC functions of π^- (solid lines) depends strongly on the momentum of the pair, *i.e.*, it varies between the extremes of an exponential and a Gaussian form. On the contrary, the functional shape of the BEC functions of thermally produced π^- and those of K^- is of almost perfect Gaussian shape, independent of the average momentum of the pair. In case of the shown transverse momentum-dependent BEC functions, the ones for thermal pions and kaons are almost identical. In the following we shall discuss only BEC functions of K^- pairs.

Let us now require for a fit to a two-particle BEC function:

(a) The selected functional shape of the fit should reproduce the entire two-particle BEC function.

(b) The selected fitting function should respond to the changes in the average momentum of the pair.

These demands ensure that a reconstruction of the original BEC function is possible through the knowledge of only the fitted parameters. In the case of kaon interferometry, (a) and (b) are easily to fulfill. The BEC functions can be fitted with a Gaussian, because their functional shape is independent of the choice of the average momentum of the kaon-pair under consideration. Therefore, we fit our results to (for the choice of the variables, *cf., e.g.* refs. [4, 12])

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + \exp\left[-q_\parallel^2 R_{long}^2(\vec{K}) - q_{side}^2 R_{side}^2(\vec{K}) - q_{out}^2 R_{out}^2(\vec{K}) - 2\,q_\parallel q_{out} R_{cross}^2(\vec{K})\right] . \quad (3)$$

A characteristic property of particle production from an expanding source is a correlation between the space-time point where a particle is emitted and its energy-momentum [13]. As a consequence, the inverse widths $R_i(\vec{K})$ ($i = \parallel, side, out, cross$) extracted from Bose-Einstein correlation functions show a characteristic dependence of the average momentum of the pair, $\vec{K}$ (*cf.* also Fig. 1).

Figs. 2a and 2b show the calculations for R_{side}, R_{out} and R_{long} as functions of rapidity y_K and transverse momentum $K_\perp$ from the fit of eq. (3) to the BEC functions in one q-dimension, *i.e.*,

the remaining momentum differences have been set equal to zero. R_{cross} has therefore not been extracted from a fit. For the specific choices of $y_K = 0$ and $K_\perp = 0$, the inverse widths of the kaon BEC from $Pb + Pb$ at 160 $AGeV$ have already been shown[1] in ref. [9]. Since measurements of kaon interferometry radii have not been published yet for this particular heavy-ion reaction, the results shown in Figs. 2a and 2b have to be considered as further predictions of the hydrodynamical analysis presented in refs. [5, 9].

In the following, we introduce an approximation [14, 15] of the original definition (*cf.* eq. (1)) of the two-particle BEC,

$$C_2(\vec{k}_1, \vec{k}_2) \approx 1 + \frac{|\int d^4x \, g_K^{dir}(x_\nu, K^\nu) \, e^{iq^\nu x_\nu}|^2}{|\int d^4x \, g_K^{dir}(x_\nu, K^\nu)|^2} \equiv \tilde{C}_2(\vec{K}, \vec{q}), \tag{4}$$

where $q^0 = E_1 - E_2$ and $E_K \equiv K^0 = \sqrt{m^2 + \vec{K}^2}$. In connection with analytical parametrizations of the momentum-dependent space-time density distribution g_K^{dir}, the approximation eq. (4) has been shown to be acceptable [4, 15, 16, 17]. Here, we are going to apply the hydrodynamical solution already presented in refs. [5, 9] for g_K^{dir} according to eq. (2).

Using $x^\nu \equiv (t, x, y, z)$, we obtain for kaon pairs with $|\vec{q}| \ll E_K$ approximately

$$q^\nu x_\nu \approx (\beta_\perp q_{out} + \beta_\parallel q_\parallel) \, t - q_{out} \, x - q_{side} \, y - q_\parallel \, z, \tag{5}$$

where $\beta_i = K_i/E_K$ ($i = \parallel, \perp$). For any cylindrically symmetric emission function which can be roughly expressed in Gaussian form one finds with eq. (5) when expanding $\exp[iq^\nu x_\nu]$ in eq. (4) for $q^\nu x_\nu \ll 1$ the expression

$$\tilde{C}_2(\vec{K}, \vec{q}) \approx 1 + \{1 \quad - \quad q_{side}^2 \langle y^2 \rangle - \langle [q_{out}(\beta_\perp t - x) + q_\parallel(\beta_\parallel t - z)]^2 \rangle$$

$$+ \quad \langle q_{out}(\beta_\perp t - x) + q_\parallel(\beta_\parallel t - z) \rangle^2 + \mathcal{O}[(q^\nu x_\nu)^4]\}, \tag{6}$$

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[1] Note the reduction of the inverse widths by a factor $1/\sqrt{2}$. The inverse widths of ref. [9] have been obtained with a different parametrization.

where

$$\langle \xi \rangle \;\equiv\; \langle \xi \rangle (k^\nu) \;=\; \frac{\int d^4x \; \xi \; g_K^{dir}(x_\nu, k^\nu)}{\int d^4x \; g_K^{dir}(x_\nu, k^\nu)} \;. \tag{7}$$

The average $\langle \xi \rangle$ can be considered as an expectation value from space-time densities g_K^{dir}. After exponentiating eq. (6), for any cylindrically symmetric system the BEC function $\tilde{\mathcal{C}}_2$ can be expressed for small momentum differences ($q_i R_i \ll 1$) in the form of eq. (3). When comparing the coefficients of the $q_i q_j$ terms, the functions $R_i^2(\vec{K})$ are found to be given through (*cf.* [16] and refs. therein)

$$
\begin{aligned}
R_{side}^2(\vec{K}) &\approx \langle y^2 \rangle = \langle y^2 \rangle - \langle y \rangle^2 \equiv \sigma_y^2 \,, \\
R_{out}^2(\vec{K}) &\approx \langle (x - \beta_\perp t)^2 \rangle - \langle x - \beta_\perp t \rangle^2 \equiv \sigma_{x - \beta_\perp t}^2 \,, \\
R_{long}^2(\vec{K}) &\approx \langle (z - \beta_\| t)^2 \rangle - \langle z - \beta_\| t \rangle^2 \equiv \sigma_{z - \beta_\| t}^2 \,, \\
R_{cross}^2(\vec{K}) &\approx \langle (x - \beta_\perp t)(z - \beta_\| t) \rangle - \langle x - \beta_\perp t \rangle \langle z - \beta_\| t \rangle \equiv \sigma_{x - \beta_\perp t,\, z - \beta_\| t} \,.
\end{aligned}
\tag{8}
$$

Thus the inverse widths of the two-kaon BEC function can be calculated directly from the ten space-time averages $\langle t \rangle$, $\langle x \rangle$, $\langle z \rangle$, $\langle t^2 \rangle$, $\langle x^2 \rangle$, $\langle y^2 \rangle$, $\langle z^2 \rangle$, $\langle xt \rangle$, $\langle zt \rangle$ and $\langle xz \rangle$ (note that $\langle y \rangle \equiv 0$ for a cylindrically symmetric expanding system). The quantities σ_i^2 and $\sigma_{i,j}$ in eqs. (8) can be interpreted as the variance (*cf.* also ref. [18]) of the random variable i and the covariance of the random variables i and j, respectively (of course, $x^\nu = (t, x, y, z) \in \Sigma$). The inverse widths of kaon BEC functions have therefore a geometrical interpretation involving the relativistic kinematics of the emitted particle.

In Figs. 2a and 2b the space-time averages from space-time densities have been calculated from the r.h.s of eqs. (8) and are compared to the inverse widths of kaon BEC functions obtained from the direct fit (3). The agreement is remarkable, although R_{side} and R_{out} from fits to kaon BEC functions are always slightly underestimated, while R_{long} is always slightly overestimated. The differences of R_{long}, R_{side} and R_{out} of both calculations have their origin in the approximations (4) - (6) and in the fact that the kaon BEC functions are not of perfect Gaussian shape for all average momenta of the kaon pairs. The maximal differences are $0.54 \; fm$, $0.15 \; fm$ and $0.17 \; fm$, when comparing R_{long}, R_{side} and R_{out} of both calculations, respectively. If one is willing to accept

errors of the magnitude here presented, one is tempted to avoid the calculation and subsequent fit of two-kaon BEC functions, since the inverse widths $R_i(\vec{K})$ can be calculated in a much more effortless way directly from space-time averages. The feature of BEC of identical kaons to be of Gaussian shape leads through the knowledge of the functions $R_i(\vec{K})$ always to an acceptable reconstruction of single BEC functions. In addition to the inverse widths which have been extracted from BEC functions of kaons, the cross-term radius $R_{cross}(\vec{K})$ has been calculated from the space-time expectation values (*cf.* Fig. 2b).

Let us now return to the BEC functions of identical pions. We have found that the inverse widths of BEC of direct pions can be also treated as in the case of interferometry of identical kaons, *i.e.*, through the use of effective Gaussians. The maximal differences when comparing R_{long}, R_{side} and R_{out} of both calculations are 0.78 fm, 0.20 fm and 0.20 fm, respectively. But it is obvious from the above presentation that an effective Gaussian would not constitute an appropriate fit for a single BEC function of all pions, *i.e.*, including those from the decay of resonances, because of the drastic change in the functional shape (*cf.* Fig. 1). When it comes to the fit of experimentally observed BEC of pions, the situation is even more complicated, although measured BEC might have a Gaussian shape[2]. Experimentally obtained BEC functions have to be calculated from separate integrations of the numerator and the denominator of the correlators of BEC functions of single momentum pairs with respect to detector acceptances [5, 19].

To summarize, Bose-Einstein correlation functions of identical kaons have been calculated for $Pb + Pb$ at 160 $AGeV$ from a (3+1)-dimensional solution of the relativistic Euler-equations. From the hydrodynamical treatment it has been found that almost all kaon BEC are of Gaussian shape, irrespective of the average momenta of the kaon pairs under consideration. Since in the calculations the kaon source was assumed to be completely chaotic, the BEC of kaons have been parametrized with a Gaussian and inverse widths of the BEC functions have been extracted.
Interpreting the source function $g_K^{dir}(x_\nu, k^\nu)$ as the quantum analogue to the space-time density

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[2] It is therefore unclear, how the conclusions of ref. [18] are affected.

distribution of emitted kaons of fixed four-momentum k^ν, space-time averages of space-time coordinates of the source have been calculated and are related to the inverse widths of BEC of kaons. The comparison of inverse widths of kaon BEC functions and space-time extensions from space-time densities shows a satisfactory agreement. It is concluded that because of the Gaussian shape of the kaon correlation functions, the space-time parameters of the kaon source can be calculated directly from space-time densities within errors of several 0.1 fm.

In the case of intensity interferometry of identical pions this simplification is not recommended when applying Gaussian fits because of the presence of resonance decays the knowledge of only the inverse widths of the two-pion BEC is not sufficient to reconstruct the entire correlation function.

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Figure Captions

Fig. 1 Examples of Bose-Einstein correlation functions of all π^- (solid lines), thermal π^-, and K^- (dashed lines), for $K_\perp = 0$ and $K_\perp = 1 GeV/c$. The BEC functions have been calculated for $Pb + Pb$ at 160 $AGeV$.

Fig. 2a Momentum-dependence of R_{side} and R_{out} extracted from Bose-Einstein correlation functions of kaons compared to the directly from space-time densities calculated ones.

Fig. 2b Momentum-dependence of R_{long} extracted from Bose-Einstein correlation functions of kaons (BEC) compared to the directly from space-time densities calculated ones. The radius R_{cross} has been calculated from space-time densities only.
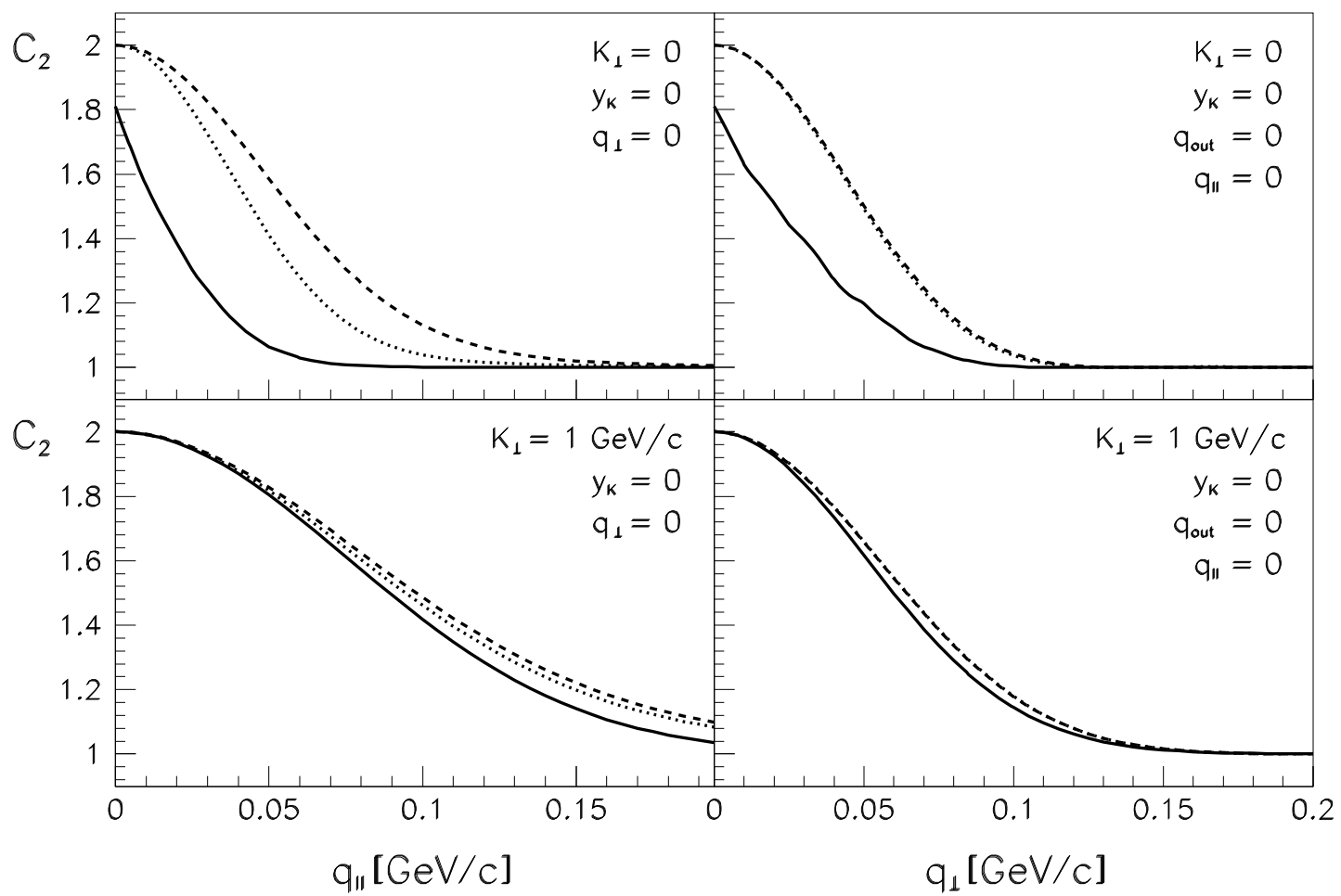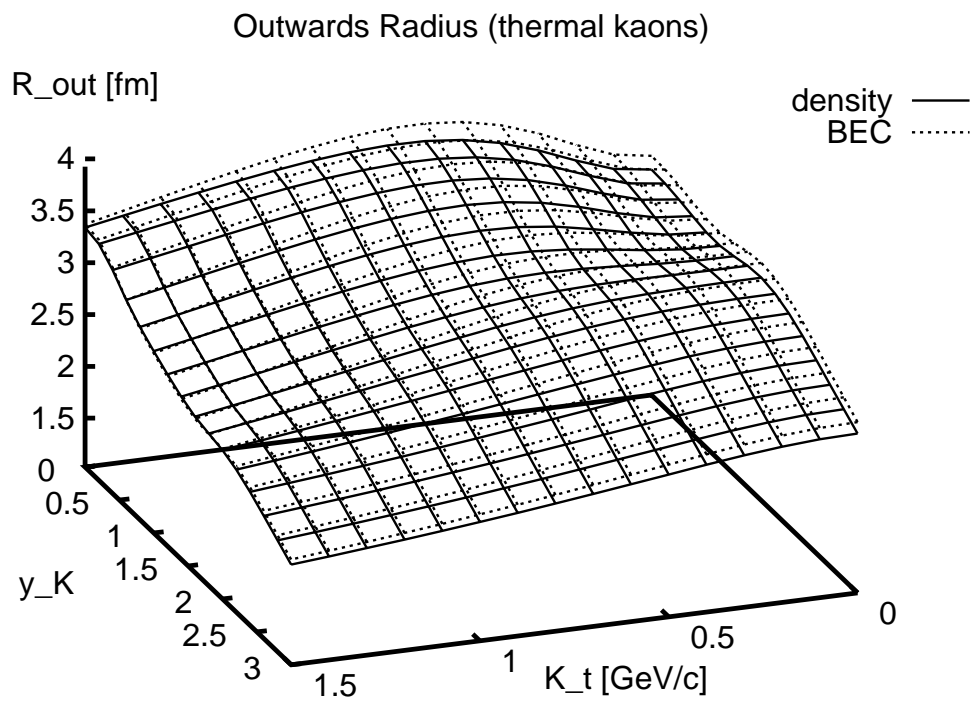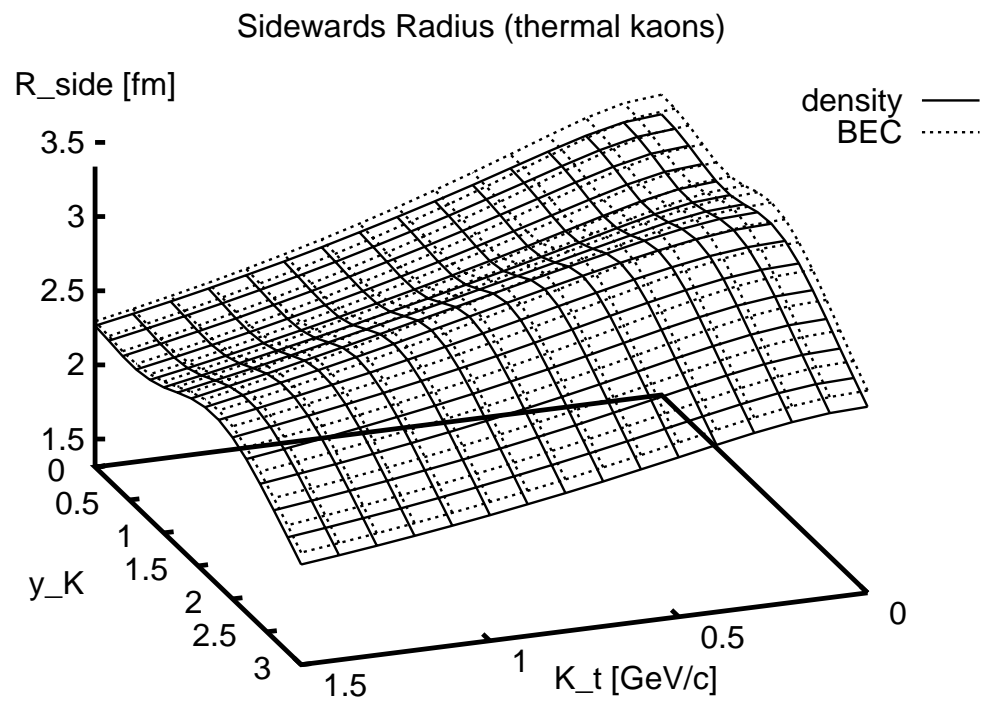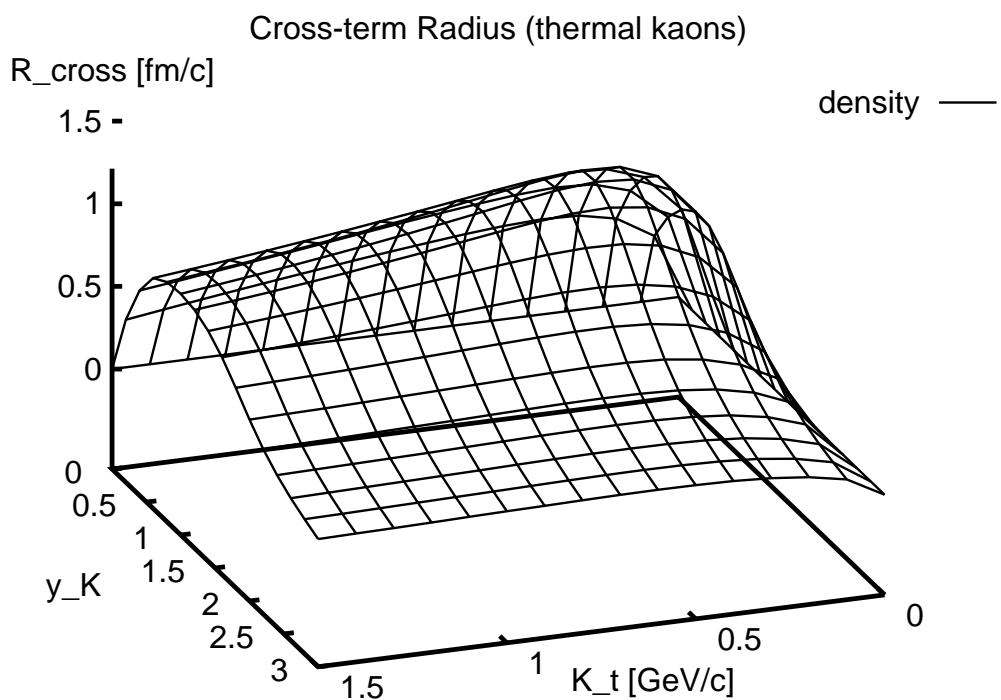
Figure 1

Sidewards Radius (thermal kaons)

R_side [fm]

density ——
BEC ·······

y_K

K_t [GeV/c]

Outwards Radius (thermal kaons)

R_out [fm]

density ——
BEC ·······

y_K

K_t [GeV/c]

Figure 2a

Longitudinal Radius (thermal kaons)

R_long [fm/c]

density ———
BEC ·········

y_K

K_t [GeV/c]

Cross-term Radius (thermal kaons)

R_cross [fm/c]

density ———

y_K

K_t [GeV/c]

Figure 2b